Midwest Theory Day  
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Invited Lecture:  

Crossing Numbers of Graphs  

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Abstract  

The crossing number of a graph $G$ is the minimal number of edge  
crossings in a drawing of $G$ in the plane. A common interior point  
(crossing) of $k$ edges contributes $\binom{k}{2}$ to this number. After giving a short review of the subject, we discuss several new results concerning this parameter. For instance, we prove that if a graph of $n$ vertices can be drawn without crossing on a torus, then its crossing number (in the plane) is at most $O(Dn)$, where $D$ denotes the maximum degree of the vertices.

What happens if we slightly change the definition, as follows? We define the degenerate crossing number of $G$ just like above, with the difference that $k$-wise crossings are now counted only once. Are we up to a surprise? For instance, does the famous crossing lemma of Leighton and Ajtai et al. remain true for this new parameter? Is it true that the degenerate crossing number of a graph with $n$ vertices and $e$ edges is always at least $\Omega\frac{e^{3/2}}{n^2}$? The answer is (perhaps) yes and no. (Joint work with Géza Tóth.)