CS 535 Homework 2  
Due: February 9 (Th), in class.

Undergraduate students, answer problems 1, 2, 3. Graduate students, answer all problems.

1. Suppose array $A$ contains $n - 1$ of the $n$ integers in the set $\{1, 2, \ldots, n\}$. We want to design an algorithm that will determine the missing number. For example, if $n = 4$ and $A = [2, 1, 4]$ is the input, the algorithm should output 3.
   a. Describe a brute force way of solving this problem. What is its running time?
   b. Now, design an $O(n)$-time algorithm for the problem. Please make sure that you explain why your algorithm works and why its running time is $O(n)$.

2. There are $n$ coins $c_1, c_2, \ldots, c_n$. Of these $n$ coins, $n - 1$ are genuine and one is fake. All the genuine coins have the same weight; the fake coin is either lighter or heavier. We have a balance beam with two large pans. Our goal is to identify the fake coin with as few weighings as possible.
   Here’s a simple procedure for finding the fake coin in the case when $n = 3$: First, using the balance beam, compare $c_1$ and $c_2$. If their weights are equal, $c_3$ is the fake coin. If their weights are not equal, compare $c_1$ and $c_3$. If their weights are equal, $c_2$ is the fake coin; otherwise, $c_1$ is the fake coin. This procedure then identifies the fake coin in 2 weighings.
   a. When $n = 9$, describe a procedure that would identify the fake coin using as few weighings as possible in the worst case.
   b. Generalize your method above to an arbitrary $n$. How many weighings are needed by your procedure?

3. Let $A$ be an array containing $n$ numbers. Define $f\text{lip}(i), 0 \leq i \leq n - 1,$ to be the operation where $A[0, 1, \ldots, i]$ is modified to $A[i, i - 1, \ldots, 0]$. For example, if $A = [20, 5, -11, 3, 7]$ and $f\text{lip}(3)$ is called, the result is the array $[3, -11, 5, 20, 7]$. Our goal is to sort $A$ using $f\text{lip}(i)$ operations only.
   a. Describe a sequence of flips for $A = [20, 5, -11, 3, 7]$ so that the entries of the final array is sorted in increasing order.
   b. Now, describe a general method for sorting an array $A$ containing $n$ numbers using flips.
   c. In the worst case, how many flips will your algorithm perform? How much time will it take to implement $f\text{lip}(i)$? Based on your answers to the previous two questions, what is the running time of your algorithm?

4. C-1.28.