Everyone is required to solve problems 1, 3 and 4. Problem 2 is a bonus.

1. Write a pseudocode that solves problem 2 in the handout given in class. Make sure that the output of your pseudocode is an optimal plan, and not just the value of an optimal plan.

2. Using the matrix chain multiplication algorithm discussed in class, determine the best way of multiplying a chain of matrices $A_1, A_2, A_3, A_4, A_5$ whose dimensions are $10 \times 5, 5 \times 2, 2 \times 20, 20 \times 12, 12 \times 4$? Show your work please!

3. The maximum subsequence sum problem – again! Recall that in the maximum subsequence sum problem, we are given a sequence of $n$ numbers $x_1, x_2, \ldots, x_n$. Our goal is to find a contiguous subsequence whose sum is as large as possible. I’ve shown you four different ways to solve the problem – brute force (takes $O(n^3)$ time), improved brute force (takes $O(n^2)$ time), a “clever” algorithm (takes $O(n)$ time), and divide and conquer (takes $O(n \log n)$ time).
   a. This time around, please design a dynamic programming algorithm for this problem that runs in $O(n)$ time. Note: you must output the actual subsequence with the best sum. (Hint: For $j = 1, \ldots, n$, define $SUM(j)$ as the largest sum of a subsequence that ends at position $j$. Find a recursive formula for $SUM(j)$.)
   b. Please apply your algorithm to the sequence 5, 15, −30, 10, −5, 40, 10, −8.

4. Planning a trip. You are going on a long trip. You start on the road at mile post 0. Along the way there are $n$ hotels, at mile posts $a_1 < a_2 < \ldots < a_n$, where each $a_i$ is measured from the starting point. The only places you are allowed to stop are at these hotels, but you can choose which of the hotels you stop at. You must stop at the final hotel (at distance $a_n$), which is your destination.
   You’d ideally like to travel 200 miles a day, but this may not be possible because of the spacing of the hotels. If you travel $x$ miles during a day, the penalty for that day is $(200 - x)^2$. Notice that there is a penalty when you travel less than 200 miles or when you travel more than 200 miles. You want to plan your trip so as to minimize the total penalty – i.e., the sum of the daily penalties over all travel days.
   a. If you’re me, you’d consider this greedy algorithm to solve the problem: On the first day, choose the hotel at $a_i$ so that $(200 - a_i)^2$ is as small as possible. On the second day, choose the hotel at $a_j$ where $a_j > a_i$ so that $(200 - (a_j - a_i))^2$ is as small as possible. Keep doing this until the hotel at $a_n$ is reached. Show that this greedy algorithm does not work. That is, find an example that show that this algorithm will not produce an optimal plan.
b. Design a dynamic programming algorithm for this problem. Once again, you must output the actual plan. (*Hint: For \( j = 1, \ldots, n \), define \( \text{PENALTY}(j) \) to be the largest penalty incurred by a trip that considers only hotels 1, \ldots, \( j \).*)