CS 535 Homework 2  
Due: September 24 (W), in class.

Undergraduate students, answer problems 1, 2, 3. Graduate students, answer all problems. Whenever you present an algorithm, make sure that you argue its correctness and analyze its running time.

1. C-1.28

2. You must have heard about the UWM’s new connection to Microsoft – their new CEO Satya Nadella is a UWM alumnus. He received his Master’s in Computer Science from UWM in 1990. This problem will be your new connection to Bill Gates! Bill Gates has published only one research paper and it’s on the burnt pancake problem.

A diner serves a breakfast plate called High Towers which simply consists of $n$ pancakes. A very bad cook fills in the order. He proceeds by making $n$ pancakes – all of different sizes and all with one side burnt. He then piles the pancakes onto a plate to create a tower of pancakes. Mister Waiter picks up the plate and immediately realizes that he has to make the plate presentable. He wants to (i) sort the pancakes from smallest to largest with the largest pancake at the bottom of the plate, and (ii) hide the burnt side of each pancake. Now he only has one move available to him – push a spatula underneath one of the pancakes and then flip the entire subtower of pancakes above the spatula upside down. His goal is to use this move repeatedly until his goals are accomplished. How many moves does he need?

Let’s create a mathematical model for this problem. Represent the $i$th smallest pancake as $i$ if the pancake has its good side facing up and as $-i$ if its burnt side is facing up. Let the array $P[1, \cdots, n]$ store the pancakes on the plate created by the bad cook. In particular, $P[j]$ refers to the $j$th pancake from the top. Thus, for $n = 5$ the input array can look like this: $P = [-3, 2, 4, -1, -5]$.

Let $\text{flip}(i)$, $1 \leq i \leq n$, be the operation where the entries of $P[1 \cdots i]$ is modified as $[-P[i], -P[i-1], \ldots, -P[1]]$. For example, if we apply $\text{flip}(3)$ to the above example, the results is $[-4, -2, 3, -1, -5]$. Our goal is to transform the input $P$ into the array $[12 \cdots n]$ using $\text{flip}$ operations only.

a. Describe a sequence of flips that transforms $P = [-3, 2, 4, -1, -5]$ into $[1, 2, 3, 4, 5]$.

b. Now write a pseudocode that transforms an arbitrary input $P$ of $n$ numbers into $[1, 2, \cdots, n]$ using $\text{flip}$ operations only.

c. How much time does it take to implement $\text{flip}(i)$? In the worst case, how many flips will your algorithm perform? Based on your answers to these previous two questions, what is the running time of your algorithm?
3. There are \( n \) coins \( c_1, c_2, \ldots, c_n \). Of these \( n \) coins, \( n - 1 \) are genuine and one is fake. All the genuine coins have the same weight; the fake coin is either lighter or heavier. We have a balance beam with two large pans. Our goal is to identify the fake coin with as few weighings as possible.

Here’s a simple procedure for finding the fake coin in the case when \( n = 3 \): First, using the balance beam, compare \( c_1 \) and \( c_2 \). If their weights are equal, \( c_3 \) is the fake coin. If their weights are not equal, compare \( c_1 \) and \( c_3 \). If their weights are equal, \( c_2 \) is the fake coin; otherwise, \( c_1 \) is the fake coin. This procedure then identifies the fake coin in 2 weighings.

a. When \( n = 9 \), describe a procedure that would identify the fake coin using as few weighings as possible in the worst case.

b. Generalize your method above to an arbitrary \( n \). How many weighings are needed by your procedure?

4. In the maximum subsequence sum problem, we were interested in finding the subsequence that had the largest sum. Consider the two dimensional version of this problem. Given an \( n \times n \) matrix whose entries are integers. Our goal is to find a submatrix (not necessarily square) whose sum is the largest. Write a pseudocode that solves this problem in a brute force manner (i.e., it considers all possible submatrices). What is its running time of your algorithm?