

Metric inequalities for polygons

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Abstract

Let A_1, A_2, \dots, A_n be the vertices of a polygon with unit perimeter, that is $\sum_{i=1}^n |A_i A_{i+1}| = 1$. We derive various tight estimates on the minimum and maximum values of the sum of pairwise distances, and respectively sum of pairwise squared distances among its vertices. Such estimates on these sums in the literature were known only for convex polygons. We also sharpen a previous lower bound on the minimum sum of pairwise squared distances for convex polygons due to Novotný.

In the second part, we determine an exact formula for the maximum perimeter of a simple n -gon (n odd) contained in a disk of unit radius. This answers an open problem posed by Braß. We then examine what happens if the simplicity condition is dropped, and obtain an exact formula for the maximum perimeter in this case as well.

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