Homework # 4
due September 30

1 Reading

Please read Chapter 6 in your textbook. It's not necessary to understand everything here. We won't dwell on the "nameless representation" of terms in later weeks.

2 Problems

Please do the following problems from the book:

6.1.1 (p76), 6.2.2 (p79) 6.3.1 (p81).

There is no need to turn the answers in, as the answers are in the back of the book. These are all easy exercises. Do them!

3 SASyLF Proofs

Prove that our definition of substitution is "total," that is, that substitution is always defined if the arguments are valid. Also prove Exercise 6.2.6 when s is a 0-term, and t is a $j+1$-term.

Use the skeleton file to start both proofs. The skeleton file is very long because it includes a lot of helpful lemmas. Make sure you understand the lemmas, especially the ones that the skeleton file warns that you will need.

4 Discussion

1. The rule [E-AppAbs] on page 81 is written

$$(\lambda.t_{1\,2})v_2 \longrightarrow \uparrow^{-1}([0 \mapsto \uparrow^1(v_2)]t_{1\,2})$$

   This requires two shifting operations. Consider the following suggested re-definition of the rule that does only one shift:

$$(\lambda.t_{1\,2})v_2 \longrightarrow [-1 \mapsto v_2]\uparrow^{-1}(t_{1\,2})$$

   If this rule works, explain why you think it isn't used in the book. If it doesn't work, give an example where it gives a different result than the definition in the book.

2. In our formulation of substitution and evaluation of nameless terms, we use the following version of the rule with no shifts:

$$(\lambda.t_{1\,2})v_2 \longrightarrow [0 \mapsto v_2](t_{1\,2})$$

   When does this rule work? When does it not work? Explain!