Homework 4

1. BSTs: search, insert and delete
   (a) Consider the following sequence of keys: 10, 3, 8, 7, 5, 2. Can these be the keys on a search path in a BST? Why or why not?
   (b) Briefly explain how to implement insertion and deletion in a binary search tree, where no balancing mechanism is used.
   (c) Show the binary search trees that result after successively inserting the keys 6, 5, 1, 2, 8, 3, 9, 7, 4 into an initially empty binary search tree.
   (d) Do the same when deleting 6, 5, 7, 2, 3 (in this order).

2. Red-black tree insert
   Show the red-black trees that result after successively inserting the keys 6, 5, 4, 1, 3, 2, 7, 8, 9, 10 into an initially empty red-black tree. What are the height and the black-height of the resulting tree?

3. Binary search trees
   (a) Given a binary tree, give efficient algorithms to determine (i) the total number of nodes; and (ii) its height. Analyze the two algorithms.
   (b) Given a red-black tree, give efficient algorithms to determine (i) the total number of nodes; (ii) its height; and (iii) its black height. Analyze the three algorithms. Are there any differences to part (a) one can observe?

Other suggested exercises (for practice; you do not need to turn in)

4. AVL trees
   Draw an AVL tree of height 5 with the minimum number of nodes, and one with the maximum number of nodes.

5. AVL trees
   [This exercise is for graduate students.] Is there any height $h$ so that the ratio between the maximum number of nodes of an AVL tree of height $h$ and the minimum number of nodes of an AVL tree of height $h$ is at least 10? Justify your answer.