

## CS 535 Homework 7

**Due: November 4 (W), in class.**

Undergrads, please answer questions 1, 2 and 3(a and b only). Question 3c is a bonus. Grad students, please answer questions 1, 2, and all of 3. Question 4 is a bonus for everyone.

1. C-4.14
2. C-4.19
3. *Finding the weighted median.* Let  $S$  be a sequence of numbers. When  $|S|$  is odd, the *median* is the “middle” number when  $S$  is sorted; when  $|S|$  is even, the *lower and upper median* are the two “middle” numbers. In other words, the median of  $S$  is a number  $x$  such that at most half of the numbers in  $S$  are less than  $x$  and at most half of the numbers in  $S$  are greater than  $x$ .

Now, suppose the numbers have weights associated with them. Let  $x_1, x_2, \dots, x_n$  be  $n$  distinct numbers with positive weights  $w_1, w_2, \dots, w_n$  so that  $\sum_{i=1}^n w_i = 1$ . Define the *weighted lower median* of the set is the number  $x_k$  that satisfies the following conditions:

$$\sum_{x_i < x_k} w_i < \frac{1}{2} \text{ and } \sum_{x_i > x_k} w_i \leq \frac{1}{2}.$$

That is, in the weighted version, the lower median of  $S$  is a number  $x$  such that the total weight of all numbers less than  $x$  is at less than 0.5 (half of the total weight of all the numbers) and the total weight of all numbers greater than  $x$  is less than or equal to 0.5.

- a. Suppose the numbers are 1, 2, 3, 4, 5 and the corresponding weights of the numbers are 0.6, 0.1, 0.1, 0.1, 0.1. What is the weighted lower median of these numbers?
- b. Using sorting, show how to compute the weighted lower median of the  $n$  elements in  $O(n \log n)$  worst-case time.
- c. Using an algorithm like QuickSelect, show that the weighted lower median of the  $n$  elements can be found in  $O(n)$  average time.
4. *Finding the median of two sorted arrays.* Let  $X[1 \dots n]$  and  $Y[1 \dots n]$  be arrays each containing  $n$  numbers already in sorted order. Give an  $O(\log n)$ -time algorithm that finds the lower and upper median of all  $2n$  elements. Please explain why your algorithm works!