CS 535 Homework 3
Due: September 30(W), in class.

There are four problems in this homework, all about designing algorithms using certain ADT's. If you are registered as an undergraduate student, please answer questions 1, 2 and 3. Question 4 is a bonus problem. If you are registered as a graduate student, 1, 2 and 4. Question 3 is a bonus problem.

1. *Lists.* Suppose L_1 and L_2 are two LIST ADT's whose elements are sorted in increasing order. Assume that a doubly linked list was used to implement a LIST ADT.

   a. Describe an algorithm that outputs in sorted order the elements that L_1 and L_2 have in common (i.e., the elements of $L_1 \cap L_2$). What is the runtime of your algorithm?

   b. This time, describe an algorithm that outputs in sorted order all the elements that appear in L_1 or in L_2 (i.e., the elements of $L_1 \cup L_2$). What is the runtime of your algorithm?

2. C-2.9. Note that you should not use a tree traversal algorithm to answer the questions. For example, running a preorder traversal of T to determine $preorderNext(v)$ is not considered a legitimate answer.

3. Consider the following instance of the stable marriage problem.

m_1:	w_5	w_7	w_1	w_2	w_6	w_8	w_4	w_3
m_2:	w_2	w_3	w_7	w_5	w_4	w_1	w_8	w_6
m_3:	w_8	w_5	w_1	w_4	w_6	w_2	w_3	w_7
m_4:	w_3	w_2	w_7	w_4	w_1	w_6	w_8	w_5
m_5:	w_7	w_2	w_5	w_1	w_3	w_6	w_8	w_4
m_6:	w_1	w_6	w_7	w_5	w_8	w_4	w_2	w_3
m_7:	w_2	w_5	w_7	w_6	w_3	w_4	w_8	w_1
m_8:	w_3	w_8	w_4	w_5	w_7	w_2	w_6	w_1

w_1:	m_5	m_3	m_7	m_6	m_1	m_2	m_8	m_4
w_2:	m_8	m_6	m_3	m_5	m_7	m_2	m_1	m_4
w_3:	m_1	m_5	m_6	m_2	m_4	m_8	m_7	m_3
w_4:	m_8	m_7	m_3	m_2	m_4	m_1	m_5	m_6
w_5:	m_6	m_4	m_7	m_3	m_8	m_1	m_2	m_5
w_6:	m_2	m_8	m_5	m_3	m_4	m_6	m_7	m_1
w_7:	m_7	m_5	m_2	m_1	m_8	m_6	m_4	m_3
w_8:	m_7	m_4	m_1	m_5	m_2	m_3	m_6	m_8

   a. Find a stable matching of the instance using the Gale-Shapley algorithm. The result is the *man-optimal* stable matching.

b. This time, find the *woman-optimal* stable matching by switching the roles of the men and women in the algorithm.

c. This instance turns out to have more than two stable matchings. Find one that is different from the man-optimal and woman-optimal stable matching. Make sure that you argue why your matching is in fact stable.

4. Again, consider an arbitrary stable marriage instance. Suppose μ_1 and μ_2 are two of its stable matchings. Let us create a new set of man-woman pairs as follows: for each man m, match him to $better(p_1(m), p_2(m))$, where $p_i(m)$ is his partner in μ_i for $i = 1, 2$ and $better$ chooses the one he prefers more. Let us call this new set μ_{better}.

   a. Show that in μ_{better} no woman is matched to more than one man. Why does this imply that μ_{better} is a perfect matching?

   b. Show that μ_{better} contains no instabilities. Hence, μ_{better} is another stable matching.