Name: ______________________________ October 14, 2008

CS 317 Exam I

Instructions: There are six problems in this exam. Some of them consist of several subproblems. Make sure you look through them carefully. Please write legibly and justify all your answers.

1. Show that $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$ using (a) truth tables and (b) propositional identities. (Hint: Keep in mind that $r \vee r \equiv r$. )

2. Consider the following statements:

   "Somebody in the class enjoys whale watching."

   "Every person who enjoys whale watching cares about ocean pollution."

   "There is a person in this class who cares about ocean pollution."

   Let $S(x)$ denote the predicate "$x$ is a student in this class", $W(x)$ the predicate "$x$ enjoys whale watching", $P(x)$ the predicate "$x$ cares about ocean pollution". Let the domain of $x$ consist of all people.

   a. Please rewrite each of the statements above in terms of predicates $S(x), W(x), P(x)$.

   b. Using the rules of inference, explain why the third statement is a valid conclusion from the first two statements.

3. *True or false.* If your answer is true, please argue the correctness of the statement; if it is false, please provide a counterexample. Let $A$, $B$ and $C$ be sets.

   a. If $A \subseteq B$ then the power set of $A$ is a subset of the power set of $B$.

   b. If $A \cup C = B \cup C$ then $A = B$.

   c. $(A - C) \cup (B - C) = (A \cup B) - C$.

4. Suppose $A, B$ and $C$ are sets and $f : A \to B$ and $g : B \to C$ are functions. Prove the following:

   a. If $f$ and $g$ are one-to-one functions, so is $g \circ f$. (That is, show that if $a_1, a_2 \in A$ and $a_1 \neq a_2$ then $g \circ f(a_1) \neq g \circ f(a_2)$.)

   b. If $f$ and $g$ are onto functions, so is $g \circ f$. (That is, show that for every element $c \in C$, there is an $a \in A$ so that $g \circ f(a) = c$.)

   c. Thus, (a) and (b) implies that if $f$ and $g$ are bijections, so is $g \circ f$. What is $(g \circ f)^{-1}$ in terms of $f^{-1}$ and $g^{-1}$?

5. Let $C$ be a set of candidates running for an election. We would like to describe your preferences for these candidates as a relation. In particular, $R$ is a relation on $C$ such that $(c, c') \in R$ if and only if you prefer $c'$ over $c$. Please determine if $R$ is (i) reflexive, (ii) symmetric, (iii) anti-symmetric, (iv) transitive or (v) total. Please provide some explanation to your answers.

6. Let $U$ be a set and $\mathcal{P}(U)$ be the power set of $U$. Let $R$ be a relation over $\mathcal{P}(U)$ so that for any two subsets $A$ and $B$ of $U$, $(A, B) \in R$ if and only if $|A| = |B|$ (i.e., $A$ and $B$ have the same size).

   a. *Warm-up:* Suppose $U = \{1, 2, 3\}$. List all the elements of $\mathcal{P}(U)$.

   b. Now, list all the elements of $R$ that includes $\{1\}$. That is, list all pairs $(A, B)$ in $R$ such that either $A$ or $B$ equals $\{1\}$.

   c. Show that $R$ is an equivalence relation *for any* $U$. That is, your proof should work for any $U$.

   d. List all the equivalence classes of $U$ when $U = \{1, 2, 3\}$.