

1.6 Rules of Inference

Suppose we know certain propositions to be true. What can we conclude? Here's a logic problem from Martin Gardner:

Professors White, Brown and Black were lunching together. "Isn't it remarkable", said the lady, "that our names are White, Black, and Brown and one of us has black hair, one has brown hair, and one has white hair."

"It is indeed", answered the one with the black hair as Professor Black bit into his sandwich, "and have you noticed that not one has hair color to match our name?"

The lady's hair is not brown. What is the color of Professor Black's hair?

Some rules of inference

When we reason, we say that **if** certain facts or premises are true **then** certain statements must be true. Here are important implications for arriving at conclusions. We can use them because they are *tautologies* – that is, they are statements which are always true.

1. Modus ponens (“mode that affirms”)

2. Modus tollens (“mode that denies”)

3. Hypothetical syllogism

4. Disjunctive syllogism

5. Resolution

6. Addition

7. Simplification

8. Conjunction

Exercises:

- a. It is not sunny and it is colder than yesterday.
We will go swimming only if it is sunny.
If we do not go swimming, then we will take a canoe trip.
If we take a canoe trip, then we will be home by sunset.

From these premises, what can you conclude?

First, let us represent the premises as logical expressions. Let p = “It is sunny.”
 q = “It is colder than yesterday.” r = “We will go swimming.” s = “We will take a canoe trip.” t = “We will be home by sunset.”

Second, write each premise in terms of p, q, r, s, t and logical operators.

Finally, apply the rules of inference.

- b. If Superman is able and willing to prevent evil, he would do so.
If Superman is unable to prevent evil, he would be ineffective.
If he is unwilling to prevent evil, he would be malevolent.
If Superman exists, he is neither ineffective nor malevolent.
Superman does not prevent evil.

Argue that Superman does not exist.

Let p = "Superman is able to prevent evil." q = "Superman is willing to prevent evil." r = "Superman prevents evil." s = "Superman is ineffective."
 t = "Superman is malevolent." u = "Superman exists."

Write each premise in terms of p, q, r, w, t, u and logical operators.

Now, apply the rules of inference.

Rules of inference for quantified statements

1. Universal instantiation

2. Universal generalization

3. Existential instantiation

4. Existential generalization

Exercises cont'd:

- d. A student in the class has not read the book.
Everyone in the class passed the first exam.

What can you conclude?

One more example: mathematical induction

Let the domain of x be the set of positive integers. Let $P(x)$ be some propositional function about x . Suppose we know the following to be true:

1. $P(1)$
2. For all positive integers x , $P(x) \rightarrow P(x + 1)$.

What can you conclude? Why?

Exercise. Show that for any positive integer n ,

$$1 + 2 + \dots + n = n(n + 1)/2.$$

Fallacies, valid vs. sound arguments

Fallacies are statements that arise from incorrect arguments. Here are two common ones:

1. Affirming the conclusion:

2. Denying the hypothesis:

Affirming the conclusion and denying the hypothesis are examples of *invalid* arguments. Sometimes, however, it is possible to make valid arguments but the conclusion still does not make sense. For example:

All dogs have eight legs.
Jack is a dog.
Therefore, Jack has eight legs.

What's the problem? In this case, we say that the argument is not *sound*.