1.5 Rules of Inference

Suppose we know certain propositions to be true. What can we conclude? Here's a logic problem from Martin Gardner:

Professors White, Brown and Black were lunching together. "Isn't it remarkable", said the lady, "that our names are White, Black, and Brown and one of us has black hair, one has brown hair, and one has white hair."

"It is indeed", answered the one with the black hair as Professor Black bit into his sandwich, "and have you noticed that not one has hair color to match our name?"

The lady's hair is not brown. What is the color of Professor Black's hair?

Some rules of inference

When we reason, we say that **if** certain facts or premises are true **then** certain statements must be true. Here are important implications for arriving at conclusions. We can use them because they are *tautologies.*

1. Modus ponens ("mode that affirms")

2. Modus tollens ("mode that denies")

3. Hypothetical syllogism

4. Disjunctive syllogism

5. Resolution

6. Addition

7. Simplification

8. Conjunction

Exercises:

a. It is not sunny and it is colder than yesterday.
   We will go swimming only if it is sunny.
   If we do not go swimming, then we will take a canoe trip.
   If we take a canoe trip, then we will be home by sunset.

   From these premises, what can you conclude?

b. If you send me an e-mail message, I will finish writing the program.
   If you do not send me an e-mail message, I will go to sleep early.
   If I go to sleep early, I will wake up feeling refreshed.

   From these premises, why can we conclude?

Rules of inference for quantified statements

1. Universal instantiation

2. Universal generalization

3. Existential instantiation

4. Existential generalization

Exercises cont'd:

d. A student in the class has not read the book.
   Everyone in the class passed the first exam.

   What can you conclude?

One more example: mathematical induction

Let the domain of x be the set of positive integers. Let $P(x)$ be some propositional function about x. Suppose we know the following to be true:

1. $P(1)$
2. For all positive integers x, $P(x) \rightarrow P(x+1)$.

What can you conclude? Why?

Exercise. Show that for any positive integer n,

$$1 + 2 + \ldots + n = n(n+1)/2.$$

Fallacies, valid vs. sound arguments

Fallacies are statements that arise from incorrect arguments. Here are two common ones:

1. Affirming the conclusion:

2. Denying the hypothesis:

Affirming the conclusion and denying the hypothesis are examples of *invalid* arguments. Sometimes, however, it is possible to make valid arguments but the conclusion still does not make sense. For example:

> All dogs have eight legs.
> Jack is a dog.
> Therefore, Jack has eight legs.

What's the problem? In this case, we say that the argument is not *sound*.