Name: ______________________________ March 10, 2009

CS 317 Exam I

Instructions: There are five problems in this exam. Some of them consist of several subproblems. Make sure you look through them carefully. Please write legibly and justify all your answers.

1. This question has two parts. Let p, q, r, s and t be propositions.

   a. Show that $[(u \wedge t) \wedge (u \rightarrow p)] \rightarrow (p \wedge t)$ is a tautology using

   (i) truth tables

   (ii) propositional identities
   (Hint: You might want to simplify $[(u \wedge t) \wedge (u \rightarrow p)]$ first before showing that the implication is a tautology.)

   b. Suppose we know that the following statements are true: q, $\neg s$, $u \rightarrow p$, $q \rightarrow (u \wedge t)$ and $(p \wedge t) \rightarrow (r \vee s)$. Using rules of inference and the additional rule you've proven in part a, argue that r is true.

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2. Let $P(m, n)$ be the statement "m divides n", where the domain of m and n is the set of all positive integers. For each of the English sentences below, please (i) express it in terms of $P(m, n)$ together with some quantifiers and (ii) determine if it is true or false. Note that when we use "number", we mean that it is a positive integer.

   a. 1 divides every positive integer.

   b. Every positive integer always divides some number.

   c. There is a number that every positive integer cannot divide.

   d. No positive integer other than 1 can divide both 5 and 7.

3. Consider the following statement: "If x and y are odd numbers then xy is an odd number."

   a. Please state the contrapositive of this statement.

   b. Please state the converse of this statement.

   c. Please prove the statement.

   d. Is its contrapositive true? How about its converse?

4. Let A, B and C be sets. For each item below, determine if the statements are true or false. If true, please provide a general argument; if false, please provide a counterexample.

   a. $(A - B) - C = (A - C) - B$

   b. If $\mathcal{P}(A) = \mathcal{P}(B)$ then $A = B$.

   c. If $A \neq B$, then there is some element $a \in A$ so that $a \notin B$.

5. Let U be a set with n elements. When representing a subset of U, it is sometimes more convenient to use a bit string of length n, where the ith bit is set to 1 if the ith element of U is in the subset and 0 otherwise. For example, suppose $U = \{1, 2, 3, 4, 5\}$. The subset $\{1\}$ is represented by the bit string 10000 while the subset $\{2, 5\}$ is represented by 01001. In general, if $U = \{1, 2, \ldots, n\}$ we define a function

$$f : \mathcal{P}(U) \rightarrow \text{ set of all bit strings of length } n \ ,$$

such that $f(S) = x_1 x_2 \ldots x_n$ where $x_i = 1$ if $i \in S$ and $x_i = 0$ if $i \notin S$.

a. *Warm-up!* What is $f(\emptyset)$? $f(U)$?

b. What is the relationship between $f(S)$ and $f(\overline{S})$? That is, how are the bit strings representing S and its complement related?

c. Now that you have a "feel" for the function f, show f is one-to-one.

d. Show f is onto.

e. State the domain and co-domain of f^{-1}. What is f^{-1}?

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