

Homework 7

1. Sec. 5.1: 40.
2. Sec. 5.3: 4, 12, 26
3. Recall that in the section on functions, we created an encoding for the paths from the point $(0, 0)$ to the point (m, n) which either went right or up at each step. In particular, we said that each path can be represented as a bit string of length $m + n$ with exactly m 0's and n 1's. In light of your answer to problem 12 in section 5.3, how many "valid" paths are there from $(0, 0)$ to (m, n) ?
4. The population of Westfield, NJ is about 35,000. If each resident has three initials, is it true that there must be at least two individuals with the same initials? Give a justification of your answer.
5. Suppose we want to use bit strings (0/1-strings) of length *at most* n to encode the 26 letters of the alphabet. What is the smallest number n that works? (This n is the maximum length of a Morse code for letters.) Again, please justify your answer.
6. Five boys and five girls attended a concert. Since the concert hall was crowded, they had to split up. Five of them sat together on row one, the other five sat together on row two. An arrangement in this situation can be described as a sequence $(x_1, x_2, x_3, x_4, x_5, y_1, y_2, y_3, y_4, y_5)$ where $(x_1, x_2, x_3, x_4, x_5)$ are the names of the kids that sat on the first row from left to right, while $(y_1, y_2, y_3, y_4, y_5)$ are the names of the kids that sat on the second row from left to right.
 - a. How many arrangements are possible?
 - b. What if all boys sat together (and so all girls sat together too)?
 - c. What if two girls, Amy and Beth, sat next to each other in the first row?
 - d. What if all boys sat together *and* Amy and Beth are next to each other in the first row?
 - e. What if all boys sat together *or* Amy and Beth are next to each other?
7. Suppose a 13-card bridge hand is drawn from a deck of 52 cards. How many ways are there to obtain
 - a. 13 cards of the same suit?
 - b. 5 spades, 4 clubs, 3 hearts, 1 diamond?
 - c. 5 cards of one suit, 4 cards of a second suit, 3 cards of a third suit and 1 card of a fourth suit?
 - d. exactly 3 aces?
 - e. 13 cards no two of which have the same number?