

## Homework 5

1. Sec. 2.3: 18 (a,b,d only), 32, 36.
2. Let  $f : A \rightarrow B$  be a function. In class, we said that if  $f$  is one-to-one and onto, then  $|A| = |B|$ . Your job is to now show the following:
  - a. If  $f$  is one-to-one and  $|A| = |B|$  then  $f$  is also onto.
  - b. If  $f$  is onto and  $|A| = |B|$  then  $f$  is one-to-one.In other words, the if two of the three statements (i)  $f$  is one-to-one, (ii)  $f$  is onto, and (iii)  $|A| = |B|$  are true, the third one has to be true as well.
3. Show that if  $f : A \rightarrow B$  is a bijection then  $f^{-1} : B \rightarrow A$  is also a bijection.
4. Let  $U$  be a set and  $\mathcal{P}(U)$  be its power set. Recall that  $\mathcal{P}(U)$  contains *all* the subsets of  $U$ .
  - a. Suppose  $U = \{1, 2, 3, 4\}$ , what is  $\mathcal{P}(U)$ ? (Hint: there should be 16 elements in  $\mathcal{P}(U)$ .)
  - b. Define the function  $f : \mathcal{P}(U) \rightarrow \mathcal{P}(U)$  as  $f(S) = \overline{S}$  for each  $S \in \mathcal{P}(U)$ . That is,  $f$  is a function that maps each subset of  $U$  to its complement. When  $U = \{1, 2, 3, 4\}$ , list the assignments of each subset of  $U$  under  $f$ .
  - c. For a general  $U$ , prove that  $f$  is a bijection. That is, argue that  $f$  is one-to-one and onto.
  - d. What is  $f^{-1}$ ?