

## Homework 5

Due 10.13.11 (Thursday)

1. Sec. 2.3: 6.
2. Problem 22 (a-c only). For each one make sure that you determine if the function is one-to-one as well as onto. That is, we're expecting two answers. Please be rigorous with your answers!

Reminder: To show that a function  $f$  is one-to-one, either (i) show if  $f(x) = f(y)$  then  $x = y$  or (ii) show its contrapositive if  $x \neq y$  then  $f(x) \neq f(y)$ . On the other hand, if you want to show that  $f$  is *not* one-to-one, then demonstrate that there are two elements in the domain that map to the same image.

To show that a function is onto, show that for every element  $y$  in the co-domain there is an element  $x$  in the domain such that  $f(x) = y$ . That is, for each  $y$  you have to specify what the corresponding  $x$  is. On the other hand, if you want to show that  $f$  is *not* onto, then demonstrate that there is an element in the co-domain that does not have a pre-image.

3. Define  $f : \{1, 2, 3, 4\} \rightarrow \{a, b, c, d\}$  where  $f(1) = d, f(2) = c, f(3) = a$  and  $f(4) = b$ . Define  $g : \{a, b, c, d\} \rightarrow \{1, 2, 3, 4\}$  where  $g(a) = 4, g(b) = 1, g(c) = 3$  and  $g(d) = 2$ .

a.  $f$  and  $g$  are clearly bijections. What is  $f^{-1}$ ? How about  $g^{-1}$ ?

b. What is  $f \circ g$ ?  $g \circ f$ ?

For both a and b, make sure you specify the domain and co-domain of the functions you're asked to define.

4. Let  $A$  be a set, and  $a \in A$ . Define  $\mathcal{P}_a$  as the set consisting of all the subsets of  $A$  that *contain* the element  $a$ . Similarly, define  $\mathcal{P}_{-a}$  as the set consisting of all the subsets of  $A$  that *do not contain*  $a$ . For this problem, we would like to answer the following question: Which has more elements  $\mathcal{P}_a$  or  $\mathcal{P}_{-a}$ ?

a. *Warm up!* Suppose  $A = \{a, b, c, d\}$ . What is  $\mathcal{P}_a$ ? How about  $\mathcal{P}_{-a}$ ?

b. By working out the example above, you should have noticed that  $|\mathcal{P}_a| = |\mathcal{P}_{-a}|$ . But this is only one example! Please argue that this is true in general by defining a function  $f$  from  $\mathcal{P}_a$  to  $\mathcal{P}_{-a}$  and showing that  $f$  is a bijection. That is, suppose  $S$  is a subset of  $A$  with  $a \in S$ . What do you want  $f(S)$  to be?