Exercises on random variables

1. (Based on Sec. 7.4: 10.) Suppose that we flip a fair coin until either it comes up tails twice or we have flipped it six times. Our goal is to determine the expected number of times we flip the coin.

   To do so, we define a random variable X, where $X(s)$ is the number of times a coin is flipped to obtain the desired result in outcome s. For example, if $s = THHT$ then $X(s) = 4$ because two tails were obtained by four flips. Notice that $X(s) \geq 2$ since at least two flips are needed to obtain two tails and $X(s) \leq 6$ since we stop when we've flipped the coin six times. In other words, $X(s)$ can be $2, 3, 4, 5$ or 6.

   a. For $i = 2, 3, 4, 5, 6$, list the outcomes in the event $(X = i)$. For example, the event $(X = 2)$ consists of only one outcome: TT.

   b. For $i = 2, 3, 4, 5, 6$, what is $P(X = i)$?

   c. Using your answer in (b), compute $E[X]$.

   d. Compute $Var[X]$.

2. (Exercise 7.4:4) A coin is biased so that the probability a head comes up is 0.6. On average, how many heads should appear when the coin is flipped 10 times?

3. It's race-car driving season, and an insurance company that works with the drivers wants to make $\$1,000$ for every policy it sells. Here's a simplified scenario as to how the company might determine the cost of a policy. A race-car driver wishes to insure his/her car for $\$50,000$. The insurance company estimates a total loss may occur with probability 0.002, a 50% loss with probability 0.01, and a 25% loss with probability 0.1. Ignoring other partial losses, it computes the expected payment P it has to make because of the losses. Then it charges the driver a premium of $P + \$1,000$ so that on average it will earn $\$1,000$. What should this premium be?

4. Suppose n people, $n \geq 3$ play the *odd person out* game to decide who will buy refereshments. The game works as follows. Everyone flips a fair coin simultaneously. If all the coins but one come up the same, the person whose coin comes up different buys the refreshments. Otherwise, the people flip the coins again and continue until just one coin comes up different from all the others. We would like to know the expected number of rounds of coin-flipping needed to decide the odd person out with n people.

   Notice that we can view a round of coin-flipping as a "success" if an odd person out is chosen on that round; otherwise, it is a "failure". The game then is interested in reaching a successful round.

   a. What is the probability of success? of failure?

   b. What is the probability that k rounds are needed for a success to show up?

   c. What is the expected number of rounds needed for the game to end?